A Coupled Knowledge Based System Using Fuzzy Optimization for Advisory Control

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A knowledge based system is described that is designed to generate on-line advice for operators regarding the proper distribution of hydrogen resources in a refinery. The system uses a coupled architecture incorporating numerical computing in a knowledge based system environment. This arrangement allows for powerful and flexible problem solving. One portion of the coupled system formulates an optimization problem that is subsequently solved by an external routine. This application is particularly concerned with uncertainty that is present in some of the constraints. To deal with this uncertainty, a fuzzy approach to the optimization is taken. A method is presented that solves the fuzzy optimization problem using standard mathematical programming techniques. The results of the fuzzy optimization allow the crisp solution to be expanded into a neighborhood of solutions that is considered acceptable. Although this work examines a specific problem, the concepts presented are general.

Introduction

The use of knowledge based systems (KBSs) for process control applications has become increasingly more common. The large amount of information that can be transmitted to the operators and the increased complexity of processes has sparked interest in the use of KBSs. The goal is to automate some of the supervisory tasks that are currently performed by the operators. Additionally, the development of powerful and reliable tools for building such systems has made the use of the technology feasible. Much research has been dedicated to developing general methods that employ KBSs for various applications.

KBSs have been built for performing many different process control functions. These range from single loop domains such as tuning, to plant wide supervisory functions such as diagnosis and planning. There is, however, overlap between the types of functions performed using traditional techniques and KBSs (Årzén, 1991). Unfortunately, this may be a product of the desire to use KBS technology, regardless of whether there is already an appropriate solution available. This remains a problem because there is no precise definition of the types of applications for which KBS solutions are superior to traditional approaches. In addition, KBSs are using more quantitative model-based information that is normally associated with numerical methods (for example, Kramer, 1987; Petti et al., 1990). A realization that the boundaries between the technologies are not very clear might result in more cooperative applications of KBSs with traditional numeric approaches.

Systems that employ both KBS techniques and numerical methods are referred to as coupled (Kitzmiller and Kowalik, 1986). These systems offer the advantages usually associated with KBSs such as heuristic sources of information, explanation facilities, and well-designed operator interfaces. Additionally, they allow problems to be solved using proven traditional methods where they are applicable. Integrating both types of computing relieves the need to force a problem to fit a particular architecture. A coupled architecture therefore offers the greatest flexibility for problem solving. The use of KBSs that are coupled with numerical optimization algorithms has been proposed for various applications associated with scheduling, design, and control (Kowalik, 1986; Georgiou and Floudas, 1990). This article is concerned with a KBS that employs an optimization routine to generate advice for operating personnel on the proper distribution of hydrogen gas within a refinery.

Although coupling a KBS to rely on numerical methods offers a powerful environment, there are still issues that require the extension of the standard numerical toolbox. Of particular concern is the issue of uncertainty. There are many different sources of uncertainty including modeling errors, noise, and

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different opinions on operating strategy. The presence of such uncertainty prevents the practical use of numerical approaches to generate a single solution to certain problems. Some method of coping with a range of possibilities is required, so that meaningful results can be generated in the presence of uncertainty. By dealing with ranges of solutions, the KBS can produce results that are more closely related to operator generated decisions. This helps to maintain operator confidence in the system.

Uncertainty in optimization problems is usually accounted for by performing a sensitivity analysis. Once the optimum is found, information from the optimization procedure is used to estimate the rate of change of the objective function in response to changes in the model parameters. Generally, this analysis is carried out by varying parameters in a one-at-a-time manner to determine the bounds at which a change of basis will occur (indicating that a new constraint has become limiting). As an extension of this type of analysis, parametric programming can be used to find the optimal solutions for changes in model parameters, which are described by a variation over a given range. Parametric programming techniques are employed in this work to solve the optimization problem over the estimated range of uncertainty (which is modeled using fuzzy sets).

Stochastic (or probabilistic) programming methods can be used to account for the statistical properties of model parameter variations. The methods used for stochastic optimization convert the problem into an equivalent deterministic problem, so that standard techniques can be applied to find the optimal solution (Rao, 1984). Stochastic programming often results in more complex deterministic problems; additionally, the probability distributions of the uncertain parameters must be known. These methods result in a solution that has little chance (or probability) of being infeasible, yet they do not directly allow for a range of solutions over the distribution of uncertainty.

Another approach to dealing with uncertainty involves the use of flexibility analysis. These methods allow for model parameter variations over a uniformly distributed interval (Friedman and Reklaitis, 1975a, b). These concepts have been extensively applied to the process design problem for linear or nonlinear problems (Grossman et al., 1983; Swaney and Grossmann, 1985a, b; Floudas and Grossmann, 1987a, b; Fichtner et al., 1990). Flexibility analysis accounts for expected and independent variations in the model parameters to ensure feasible operation over the uncertainty. Flexible solutions usually require solving an optimization problem which is of larger dimension than the original problem.

Fuzzy set theory (Zadeh, 1965) is another method of accounting for uncertainty and is utilized in this work. The theory allows varying degrees of set membership based on a membership function defined over a range of values. The membership function usually varies from 0 to 1 in a fashion similar to probability density functions. Unlike probabilities however, fuzzy sets allow the representation of many different sources of uncertainty. These sources may or may not be probabilistic in nature and are often associated with vague representations of extent (for example, high or large). Fuzzy optimization techniques have been used by various researchers (Zimmermann, 1978; Verdegay, 1982; Dubois, 1987; Dovi and Paladino, 1990) to represent uncertainty in the objective function and constraints. The uncertainty is represented by membership functions describing the parameters in the optimization model. A solution is found that either maximizes a given feasibility measure (Dubois, 1987) or represents a range of solutions (Verdegay, 1982). These techniques can be thought of as methods for generalizing and interpreting the sensitivity information available from the optimization. Fuzzy optimization is used here to characterize the neighborhood of solutions that defines the boundaries of acceptable operating states. Parametric programming methods are effectively used to solve the fuzzy optimization problem.

This article presents the application of a coupled KBS. It is designed to apply standard programming techniques so that they can be interpreted by the KBS to formulate reasonable advice for operators. The application is an advisory system for refinery personnel to maintain the proper distribution of hydrogen gas resources. The advice is generated using a mathematical model of the hydrogen flow and parametric linear programming. Constraints are formulated using fuzzy intervals to account for any inherent uncertainty. The result is an area that characterizes the best operating region, where the size of the region is dependent on the degree of uncertainty. The KBS uses this area to evaluate the current operating state and to determine whether a change in the operating policy should be recommended.

In the first section, the specific problem to be solved and the implementation of the KBS are described. This serves as motivation to extend the classical optimization to be able to deal with fuzzy constraints. The following section describes the use of fuzzy intervals for representing the uncertainty, as well as the solution to the fuzzy optimization problem. It is also shown how this solution is used to characterize a region of preferred operating states. The next section describes the application of the method to the given problem and illustrates the results for a particular set of operating conditions. The use of the fuzzy solution is shown to help improve the competence and usefulness of the advisory system. Although this article deals with a particular problem, the concept of a flexible, coupled environment and the use of fuzzy optimization techniques are general. These ideas are easily incorporated into other applications.

Application Description

This application is designed to offer advice to operating personnel regarding the best distribution of hydrogen within an oil refinery. The application uses both heuristic procedures (those followed by the operators') and mathematical programming for determining the optimal distribution. The system is built in a graphical, decision network environment that controls the various knowledge base functions. This discussion concludes with motivation for extending the optimization routine to handle fuzzy information.

Problem statement

The specific problem to be solved is to meet the needs of the hydrogen consuming units in a refinery while minimizing the hydrogen that is wasted. The solution to this problem is motivated by the high cost of makeup hydrogen. Figure 1 is a simplified schematic showing the hydrogen flow. The CRU and CCR are the catalytic reformer unit and continuous cat-

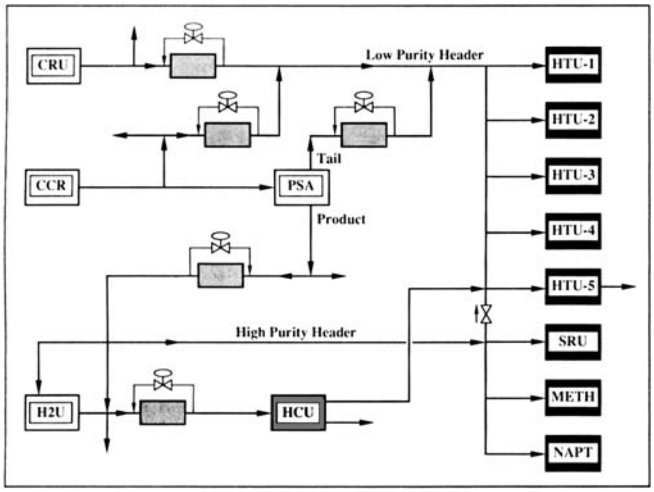

Figure 1. Hydrogen flow network.

Hydrogen producers in white, consumers in black.

alytic reformer unit, respectively. These units both produce hydrogen as by-products. The hydrocracker unit (HCU) consumes high purity hydrogen and vents low purity hydrogen. Gas from these units should be used to satisfy the needs of the hydrogen consuming hydrotreaters (HTUs), sulfur recovery (SRU), methanol (METH) and naphthalene (NAPT) units. Any additional hydrogen needs must be met by the hydrogen production unit (H2U). The pressure swing adsorption (PSA) unit separates the CCR hydrogen into a high purity product stream and a low purity tail stream. Compressors are also shown; they boost the pressure to the levels required by the consuming units. There is no facility to accumulate hydrogen.

The flow lines that dead-end in an arrow represent vent to flare or fuel gas. This is the wasted hydrogen that is to be minimized. Suction venting can be associated with each compressor if the input exceeds the current capacity. Also, the HTU-5 vent is maintained at a positive flow to control the pressure in the low purity header. There is also letdown flow from the high purity to the low purity header. These flows are monitored to determine the degree that the goal of minimal waste is met. The changes that can be made to the process include the amount of CCR hydrogen that is fed to the PSA, the amount of PSA product hydrogen that is fed to the HCU, the production rate of the H2U, and the loadings of the various compressors.

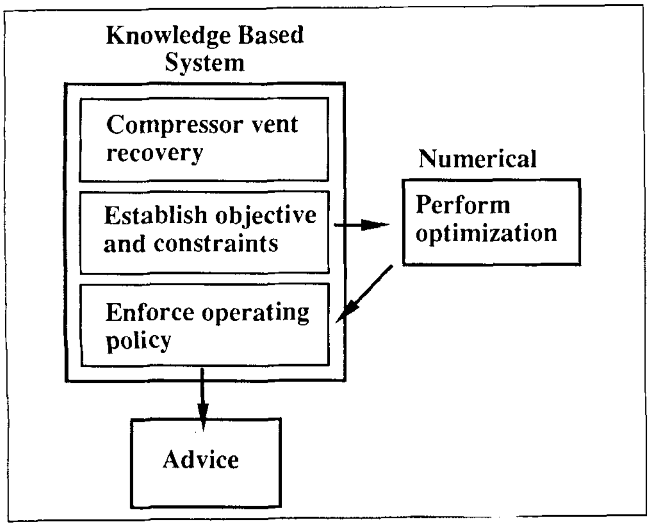

Figure 2. Architecture used by the hydrogen resource system.

The numerical algorithm solves the linear programming problem.

KBS architecture

The application solves the problem by tackling three basic subfunctions (Petti and Dhurjati, 1991):

- Attempt to recover any suction venting associated with the compressors.
- Determine the best operating policy based on the current operating state.
- Generate advice for the operators on adjustments necessary to enforce the optimal operating policy.

The architecture is illustrated in Figure 2. The KBS is a coupled system that uses heuristic information to handle the first and third subfunctions. The second subfunction employs a linear programming routine to solve an optimization problem formulated by the KBS. The methods used to implement and coordinate the subfunctions are discussed in the following section.

In the first subfunction, the system looks at the state of each compressor to determine whether suction venting is occurring. If any compressor is venting, adjustments are estimated to relieve the venting. This is accomplished using loading information and the spillback rate (flow from outlet back to inlet) of the compressor. Suction venting can usually be eliminated by increasing the loading of a compressor; however, unnecessary loading results is excessive spillback, which is a waste of power. The predicted effect on the entire hydrogen distribution is also estimated based on any recommended changes.

The main responsibility of the KBS in the second subfunction is to determine the appropriate optimization problem to be solved. Different operating configurations can be analyzed and optimization models formulated, depending on the current state of the refinery. The particular optimization model chosen by the KBS is determined on-line, and is dependent on the refinery units that are currently up and the current hydrogen distribution. The objective function can be expressed as the sum of the individual hydrogen waste flows. These objectives are commensurable and are not conflicting, so a multiobjective analysis is not warranted. The objective flows are illustrated in the simplified schematic of Figure 3, for the case when all units are operating. In order to reduce the amount of wasted hydrogen, the HTU vent should be minimized, the letdown flow should be minimized and the makeup hydrogen produced by the H2U should be minimized. The decision variables are the amount of CCR hydrogen fed to the PSA and the amount of PSA product hydrogen fed to the HCU. The constraints are given by some physical limitations as well as operator entries that describe minimum and maximum desired flows. This forms a linear optimization problem that can be solved by classical techniques (for example, the simplex method). In the implemented KBS, a heuristic optimization is also included. This heuristic approach is relied upon if the numerical optimization routine fails to return a solution.

The third subfunction is responsible for translating the re-

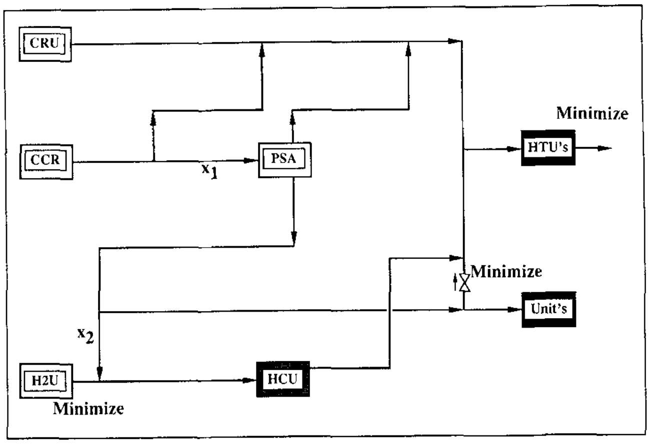

Figure 3. Optimization goals (minimize: HTU vent, letdown flow, H2U production) and decision variables.

sults of the optimization solution into physical changes to the process. The returned values of the decision variables are interpreted as changes in the PSA feed, PSA product flow to the HCU, and H2U hydrogen production rate. Additionally, the changes are analyzed to determine whether loading adjustments are required for each compressor. This is needed to ensure that the new flow rates do not cause suction venting or unreasonable spillback rates. Suggested changes to the various flow rates, controller setpoints, and compressor loadings are finally reported to the operator.

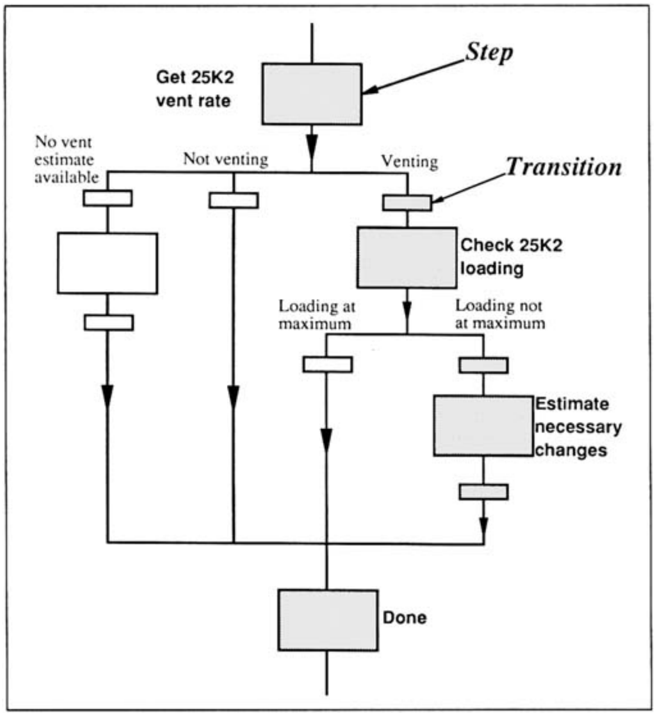

Figure 4. Grafcet network illustrating steps and transitions for compressor vent recovery.

Transition conditions must be satisfied before following steps are activated. Shaded blocks indicate the current reasoning path.

Implementation of the KBS

The KBS is developed using a toolbox built in a graphical, object-oriented environment. The toolbox forms a structured environment based on Grafcet (GREPA, 1985), a Petri-net formalism for representing sequential and parallel activities (Årzén, 1990). Each Grafcet icon is a functional object that aids in organizing the knowledge base. Since it is often difficult to maintain KBSs that are built using unstructured rules (Petti et al., 1990), the Grafcet approach ensures that the knowledge base is maintainable and that the advice can be easily reviewed by the operators. The formulas and rules that make up the knowledge base are stored on workspaces associated with Grafcet objects. These objects are connected into a network structure that represents various reasoning paths. Different conditions within the refinery result in different paths through the Grafcet network. The current path is highlighted so that the decision process can be traced to provide an explanation for the conclusions that are generated.

The Grafcet network is built using basic steps and transitions. Each step becomes active only if its associated transition is satisfied. Once activated, the step can perform various actions and analyses that are related to its function. For example, the network shown in Figure 4 is used to evaluate the suction venting of compressor 25K2. The transitions represent the various conditions that might describe the compressor, for example, venting, no venting, maximum loading, and so on. The steps following these transitions are activated according to the current state, which is determined from direct communication with the process control computer. If a step becomes active, it changes color to allow tracing of the reasoning path. Figure 4 shows that venting was detected and that the loading was not at the maximum; the shaded steps fire rules associated with recovery of this vent. Specifically, an estimate of the required loading change is calculated and the associated spillback and vent rates are revised.

In order to represent various degrees of detail, steps can be built into macrosteps as illustrated in Figure 5. These are represented as single blocks, which when activated fire a subnetwork of transitions and steps. The decision network can then be examined at the desired level of detail. Other capabilities include parallel paths, procedures, and exception transitions.

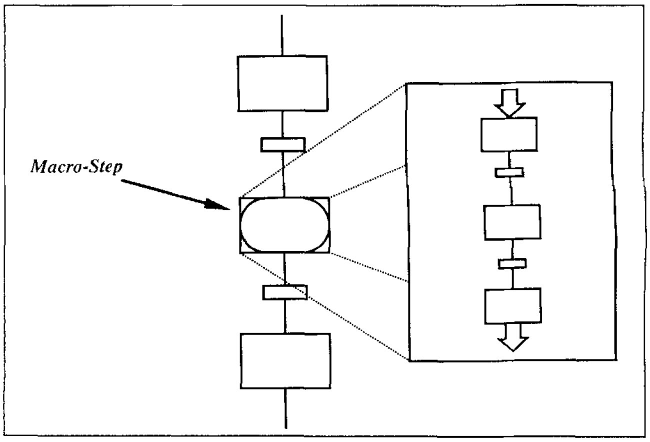

Figure 5. Grafcet macrostep.

Sub-structure becomes active when the transition to the macrostep is satisfied.

Using the various Grafcet facilities, complex networks can be built to represent the many different conditions that are possible in the refinery.

The use of the Grafcet toolbox is very helpful from both a development and operational standpoint. The various functions within the knowledge base are built in a straightforward fashion by associating rules, formulas, and procedures with Grafcet objects. This makes revision and maintenance of the system possible. Additionally, review of the reasoning path through the network is a clear indication of how the KBS arrived at its advice.

Shortcomings of using the optimal solution

The coupled KBS architecture that has been described allows an optimization routine to recommend the operating policy. This policy is based on an optimization model representing the current conditions within the refinery. The optimal solution represents the best operating state and serves as the recommended point of operation. However, the optimal solution should not be used as a firm policy for two reasons. First, there is uncertainty associated with the optimization model. Secondly, the current state of the refinery may already be sufficiently close to the optimum, particularly when considering the uncertainty that may be incorporated into the model. It is not desirable to force operating changes that are considered insignificant, or are not truly an improvement over the current policy. Continued recommendations of this kind tend to waste valuable effort and reduce the operators' confidence in the advisory system. Unfortunately, by simply using the optimal solution, there is no method of judging the appropriateness of a proposed change. It is for this reason that a fuzzy approach to optimization is examined.

In the optimization problem described in Figure 3, several of the constraints represent operators' opinions regarding desired limitations. Since opinions normally vary, the constraints on the problem are subject to some degree of uncertainty. Often some violation of the constraint within this range of uncertainty is tolerable. Limited violation of the constraints, however, may affect the solution. For instance, the current operating policy may be optimal when certain constraints are violated within their reasonable range. Changing the operating state when this is the case is to be avoided.

Some method should be included so that the optimization result can be interpreted based on an estimate of the uncertainty. The objective is to be able to characterize the appropriateness of any operating policy by comparing it to a neighborhood of solutions defined about some optimum. This neighborhood is generated by considering the optimization objective function in the presence of the estimated uncertainty in the constraints. A method is presented that satisfies this objective at low computational expense.

Fuzzy Optimization

The methods of fuzzy mathematical programming have been developed to account for uncertainty in the optimization model. These methods are essentially an extension of the sensitivity analysis that is needed to analyze variations in model parameters. Most approaches to fuzzy optimization offer a single solution to the problem that maximizes a feasibility measure defined on the constraints and the objective function (Zimmermann, 1978; Dubois, 1987). However, Ralescu (1977) argued that a fuzzy problem should have a fuzzy answer and Verdegay (1982) provides a direct method for determining the solution using parametric programming. Chanas (1983) develops a similar approach and applies it to the transportation problem (Chanas et al., 1984).

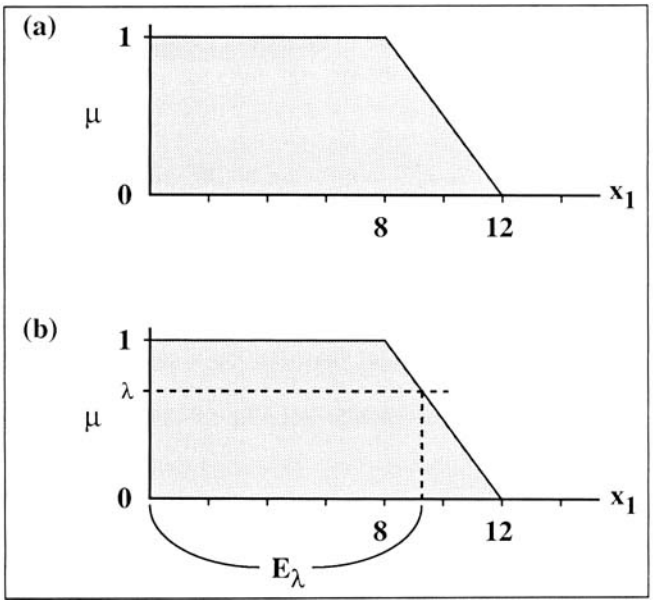

Figure 6. (a) Fuzzy constraint membership function defined on x_1; (b) λ-cut set of the fuzzy constraint.

Problem description

Although the concept of applying fuzzy sets to describe uncertainty in the optimization model can be applied to different kinds of problems, the case of a linear objective function and constraints will be considered here. Additionally, the membership functions will be assumed to be linear as well, although this restriction is not necessary to preserve the linearity of the problem (Delgado et al., 1989).

Consider the problem:

$$\begin{aligned} \text{maximize:} \quad & z = c\,x \\ \text{st:} \quad & A\,x \le b \\ & x \ge 0 \end{aligned} \tag{1}$$

where the constraints may be considered fuzzy inequalities. This type of constraint simply means that although there exists a limitation, some degree of violation of the constraint may be tolerated. For example, a fuzzy constraint on x_1 may look like that shown in Figure 6a to represent the expression,

$$x_1 \le 10 \text{ (with tolerance } \tau = 2) \tag{2}$$

The membership function ($\mu(x_1)$) spans [0, 1]; this can be interpreted as the confidence with which this constraint is satisfied (0 for low, 1 for high). Different membership functions can be defined to reflect the most appropriate representation. In general, the specific membership function represents the variations in desired operation, as expressed by different operators. A similar approach can be used to handle uncertainty in the coefficients, A and c (Verdegay, 1984), although this is not considered in this article.

Parametric solution

The fuzzy inequality constraints can be redefined in terms of their λ-cuts $\{E_\lambda \mid \lambda \in [0, 1]\}$, where

$$E_\lambda = \{\omega \mid \mu(\omega) \geq \lambda\} \tag{3}$$

see Figure 6b. The parameter λ is used to turn fuzzy inequalities into crisp inequalities. For example, Eq. 2 can be written as:

$$x_1 \leq 8 + 4(1-\lambda) \tag{4}$$

where $\lambda \in [0, 1]$.

Once all of the constraints are expressed in terms of λ, the linear programming problem can be solved parametrically (see, for instance, van de Panne, 1971). The simplex method is used here with an additional column added to account for the coefficients of λ. Additional pivots may be required if λ cannot vary in the range [0, 1] without affecting the basis. The solution is a function of λ:

$$\boldsymbol{x}^* = f(\lambda) \tag{5}$$

with the optimal value of the objective function determined by substitution into Eq. 1:

$$z^* = \boldsymbol{c}\,\boldsymbol{x}^* = g(\lambda) \tag{6}$$

This result covers all possible solutions to the optimization problem for any point in the uncertain interval of the constraints. Also note that this solution is obtained without any specialized algorithms and only increases the size of the crisp optimization problem by one variable (λ). A multiparametric problem could be formulated that allows each fuzzy constraint to vary independently with its own λ_j. However, this is not necessary if all of the constraints are parameterized consistently, thereby avoiding the increased dimensionality of such a problem.

The idea of using the λ-cuts of the fuzzy set that describes the region of feasible solutions can be used on nonlinear problems as well. The λ-cuts of the plained problem are considered, so that for each λ-cut, the classical problem obtained is solved (Verdegay, 1982). For the linear case, this is easily handled using parametric linear programming. However, other methods are required to solve nonlinear problems parametrically. If no other technique can be used, a series of problems could be formulated to generate solutions for a discrete set of λ-cuts. This can then serve as an approximation for continuous variation of λ.

The extremes ($\lambda = 0$, $\lambda = 1$) associated with the maximum and minimum values of $\boldsymbol{x}^*$, given in Eq. 5, can also be found using a flexible solution procedure (Friedman and Reklaitis, 1975a, b). However, this flexible solution requires solving a linear programming problem that is twice the size of the original crisp problem (compare this to the one additional column required by the fuzzy parametric method). Additionally, the flexible solution sacrifices the fuzzy set interpretation (λ parameter), resulting in flat intervals rather than ones with varying degrees of confidence.

Definition of the membership function

When describing the motivation for using a fuzzy optimization, the objective was stated as being able to characterize the appropriateness (or feasibility) of any operating state, $\boldsymbol{x}$. This can be accomplished using the method of Bellman and Zadeh (1970), which states that the feasibility of any decision (μ_D) is given by the intersection of the fuzzy sets describing the objective (or goal) and the constraints:

$$\mu_D(\boldsymbol{x}) = \mu_Z(\boldsymbol{x}) \wedge \mu_C(\boldsymbol{x}) \tag{7}$$

where ∧ represents the minimum operator, the usual operation for fuzzy set intersection. The value for μ_C can be easily found by intersecting the membership values for each of the m constraints:

$$\mu_C(\boldsymbol{x}) = \mu_1(\boldsymbol{x}) \wedge \mu_2(\boldsymbol{x}) \wedge \cdots \wedge \mu_m(\boldsymbol{x}) \tag{8}$$

The membership function for the objective (μ_Z) however is not as obvious. Often, predetermined aspiration values are used to define this function. Since reasonable values of this kind may not be available, the solution to the fuzzy optimization, Eq. 6, is used here to characterize this function. The idea being that larger values of z result in more confidence that the objective function is satisfied. The extremes defined by the fuzzy optimization can be used to set the bounds on the membership function:

$$\mu_Z(\boldsymbol{x}) = \begin{cases} 1 & \text{if } z(\boldsymbol{x}) \geq g(0) \\ \dfrac{z(\boldsymbol{x}) - g(1)}{g(0) - g(1)} & \text{if } g(1) \leq z(\boldsymbol{x}) \leq g(0) \\ 0 & \text{if } z(\boldsymbol{x}) \leq g(1) \end{cases} \tag{9}$$

The result is illustrated in Figure 7a in one dimension. The interpretation of this membership function is that the confidence increases as the value of the objective increases. This is reasonable because the goal is to maximize this function. The bounds on the function are defined by reasonable, attainable extremes of the objective. These are the results generated by the best and worst case values of the constraints from the fuzzy optimization.

Now that both μ_C and μ_Z have been characterized, our goal to describe the appropriateness of any operating state is realized. Given any operating policy, $\boldsymbol{x}$, the feasibility can be specified based on the objective, the constraints, and the estimated uncertainty using Eq. 7. The result in one dimension is shown in Figure 7b. The values for μ_D are shown as the minimum (intersection) of the two membership functions. Using this information, a range of feasibilities can be specified to define a neighborhood of allowable operating states, for example, any positive feasibility is acceptable. This method will be demonstrated in the following section. It may also be possible to use a more detailed representation of the membership function for control purposes, by tracking the feasibility value of the current operating state over time.

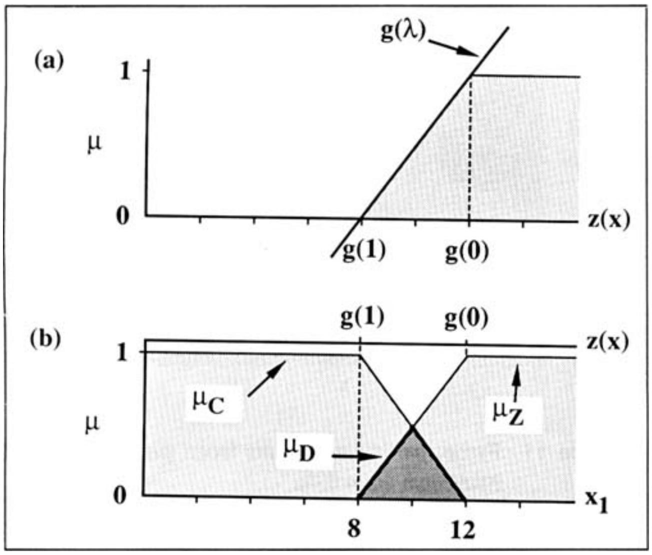

Figure 7. (a) Fuzzy set membership function defined on the objective function, z; (b) Fuzzy set membership function defining the decision region, based on the intersection.

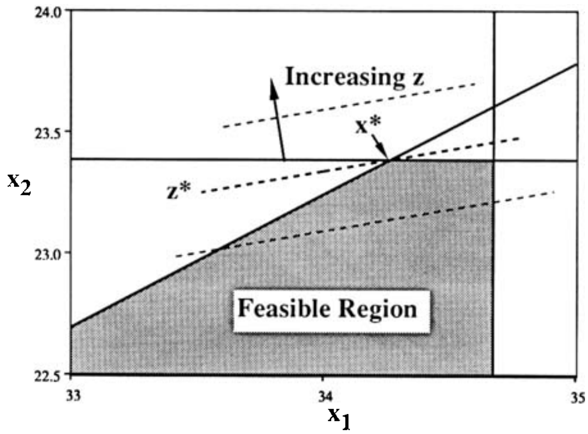

Figure 8. Solution to the crisp optimization problem.

Application to the Hydrogen Problem

The optimization problem developed for the hydrogen distribution application represents a linear programming problem in two decision variables. The variable x_1 represents the feed rate to the PSA and x_2 the rate that hydrogen is imported to the HCU (see Figure 3). This problem can be represented according to Eq. 1; the tolerance on $\boldsymbol{b}$ will be varied to demonstrate the effect of uncertainty. The optimization model for a particular operating instant, using refinery data, is defined by:

$$\boldsymbol{c} = [-0.544 \quad 3]$$
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -0.544 & 1 \end{bmatrix}$$
$$\boldsymbol{b} = \begin{bmatrix} 34.672 \\ 23.384 \\ 4.743 \end{bmatrix} \tag{10}$$

where the values given for $\boldsymbol{b}$ represent nominal values. All flow rates are in million standard cubic feet per day (MMSCFD, 1 MMSCFD = 0.3277 m^3/s at STP). Other constraints are usually specified, but these represent the ones of interest in this particular case. Some of the values used for $\boldsymbol{b}$ may be considered to be derived from operator opinion. For example, the third constraint represents the minimum letdown flow necessary to keep the valve from sticking. The value for this limit cannot be given a precise value; therefore a certain degree of violation may be tolerable. The other constraints may be subject to some uncertainty as well, as they represent the maximum allowable values for x_1 and x_2.

The solution to the crisp problem defined by Eq. 10 is shown in Figure 8. The constraints are shown, as well as a few contours of the objective function. The optimal value is given by:

$$\boldsymbol{x}^* = \begin{bmatrix} 34.267 \\ 23.384 \end{bmatrix} \tag{11}$$

with the objective at the optimum being

$$z^* = 51.511 \tag{12}$$

If the advisory KBS did not consider uncertainty, a recommendation to move to this operating point would be made, regardless of the current values of the decision variables. This is not a desirable characteristic.

If the tolerance on the third constraint is set to $\tau_3 = 0.1$, then this constraint is represented parametrically as

$$\boldsymbol{a}_3 \boldsymbol{x} \le (b_3 - \tau_3) + 2\tau_3(1-\lambda) \tag{13}$$

or

$$-0.544\,x_1 + x_2 \le 4.843 - 0.2\,\lambda$$

where $\boldsymbol{a}_3$ is the third row in $\boldsymbol{A}$ and $\lambda \in [0, 1]$. Using the procedure outlined in the previous section, the result of Figure 9 is generated. This figure shows the fuzzy set defined by the intersection of the constraints (μ_C) in the lower region. The darkest area represents where the value of $\mu_C = 1$. The lightly shaded strip indicates where the value decreases linearly, until it reaches 0 at the furthest boundary. The upper region can be interpreted similarly for μ_Z. This set (μ_Z) is defined using Eq. 9 and the fuzzy optimization solution (using Eq. 13):

$$\boldsymbol{x}^* = \boldsymbol{f}(\lambda) = \begin{bmatrix} 34.083 \\ 23.384 \end{bmatrix} + \begin{bmatrix} 0.368 \\ 0 \end{bmatrix} \lambda \quad \forall \lambda \in [0, 1] \tag{14}$$

and the optimal value of the objective function is given by:

$$z^* = g(\lambda) = \boldsymbol{c}\,\boldsymbol{x}^* = 51.611 - 0.2\,\lambda \tag{15}$$

The area of intersection between the sets defined by μ_C and μ_Z represents the membership function for μ_D. This region is shown in medium shading in Figure 9. This fuzzy set is used

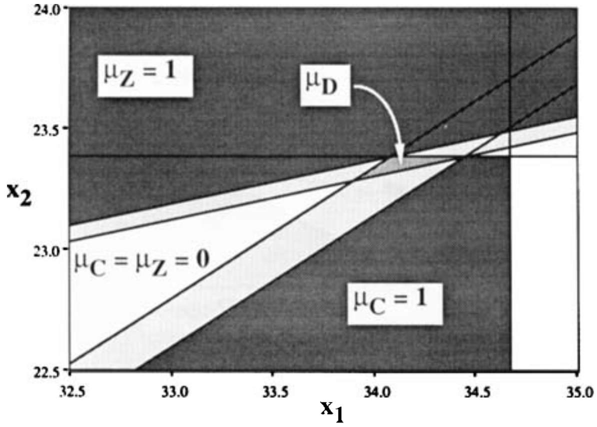

Figure 9. Fuzzy sets generated from parametric optimization ($\tau_3 = 0.1$).

Darkest area indicates $\mu = 1$, lightest shading indicates linearly decreasing, in white areas $\mu = 0$. Medium shading shows intersection of the sets (decision region).

for interpreting the current operating policy. If the state is within the intersecting region of Figure 9, then no changes are recommended. No significant improvement can be made in the value of the objective function when the uncertainty in the constraint is considered. If the current policy falls outside this region, then either one of the constraints is violated or the value of the objective function can be increased. Under these conditions, a value of x is recommended as a preferred state. A good choice for the recommended value is one that is near the center of the allowable operating region such as:

$$x^* = f(0.5) = \begin{bmatrix} 34.267 \\ 23.384 \end{bmatrix} \tag{16}$$

which corresponds to the fuzzy optimum (of for example, Zimmermann, 1978), when no basis change occurs.

To illustrate the effect of increased uncertainty on this analysis, the tolerance on the third constraint is increased to $\tau_3 = 0.2$. This results in the region shown in Figure 10. As expected, the region has increased to allow a larger range of operating states

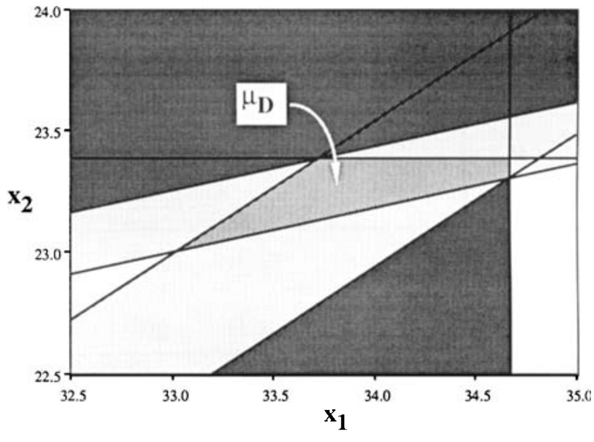

Figure 10. Fuzzy sets generated from parametric optimization ($\tau_3 = 0.2$).

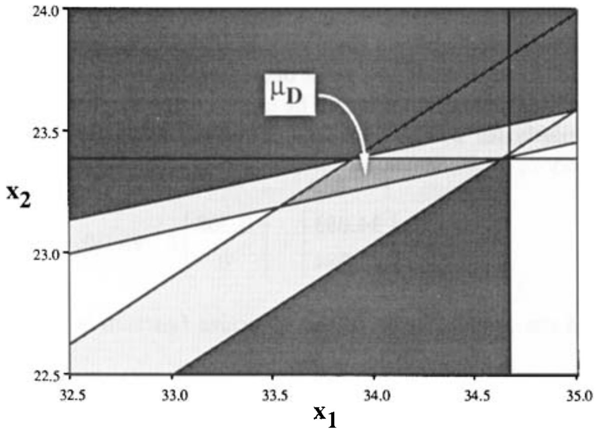

Figure 11. Fuzzy sets generated from parametric optimization ($\tau_3 = 0.3$).

New constraint becomes active.

(the medium shaded area) as compared to Figure 9. As a further example, consider the case of $\tau_3 = 0.3$. In this problem, the fuzzy optimization solution is nonlinear in λ:

$$x^* = f(\lambda) = \begin{cases} \begin{bmatrix} 33.715 \\ 23.384 \end{bmatrix} + \begin{bmatrix} 1.103 \\ 0 \end{bmatrix} \lambda & \forall \lambda \in [0,\ 0.868] \\ \begin{bmatrix} 34.672 \\ 23.905 \end{bmatrix} + \begin{bmatrix} 0 \\ -0.60 \end{bmatrix} \lambda & \forall \lambda \in [0.868,\ 1] \end{cases} \tag{17}$$

with

$$z^* = g(\lambda) = \begin{cases} 51.811 - 0.6\lambda & \forall \lambda \in [0,\ 0.868] \\ 52.852 - 1.8\lambda & \forall \lambda \in [0.868,\ 1] \end{cases} \tag{18}$$

This is caused by a change of basis as shown in Figure 11. The constraint on the maximum value of x_1 becomes active in the interval, which produces the singular point at $\lambda = 0.868$. The region generated is larger in this example but limited by the newly active constraint.

The final example uses a fuzzy value for two of the constraints. Setting the tolerance on the first constraint to $\tau_1 = 0.1$ and the third constraint to $\tau_3 = 0.3$ results in the following solution to the fuzzy optimization:

$$x^* = f(\lambda) = \begin{cases} \begin{bmatrix} 33.715 \\ 23.384 \end{bmatrix} + \begin{bmatrix} 1.103 \\ 0 \end{bmatrix} \lambda & \forall \lambda \in [0,\ 0.811] \\ \begin{bmatrix} 34.772 \\ 23.959 \end{bmatrix} + \begin{bmatrix} -0.20 \\ -0.709 \end{bmatrix} \lambda & \forall \lambda \in [0.811,\ 1] \end{cases} \tag{19}$$

and

$$z^* = g(\lambda) = \begin{cases} 51.811 - 0.6\lambda & \forall \lambda \in [0,\ 0.811] \\ 52.961 - 2.018\lambda & \forall \lambda \in [0.811,\ 1] \end{cases} \tag{20}$$

Once again there is a singular point ($\lambda = 0.811$) caused by a change in the basis. The region produced is shown in Figure 12. The area of intersection has also expanded in the direction of the uncertainty allowed by the first constraint.

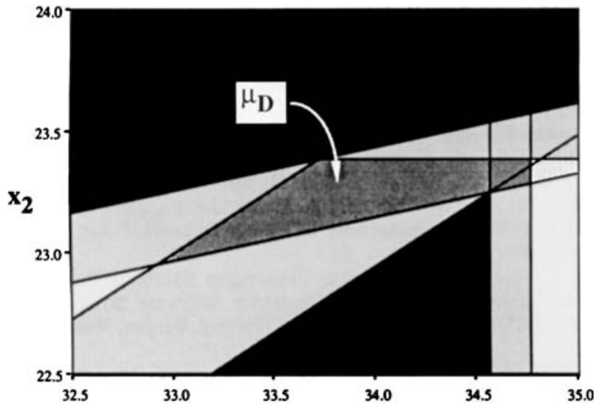

Figure 12. Fuzzy sets generated from parametric optimization ($\tau_1 = 0.1$, $\tau_3 = 0.3$).

This method for determining the allowable operating region is straightforward to formulate and the computational expense is minimal. Although some fuzzy optimization algorithms require major changes in the dimensions of the original problem, this method does not. The only additional operations over a crisp optimization (simplex method) are those associated with an extra variable and the added pivots necessary if a change of basis occurs within the range of $\lambda \in [0, 1]$ (see van de Panne, 1971).

Conclusion

KBSs show potential for handling many of the supervisory tasks within a production plant that are currently the responsibility of operators. It is argued that by using a coupled architecture, flexible systems can be built that derive power from traditional computer algorithms and knowledge based system environments. However, methods need to be developed that take advantage of this architecture and provide reasonable and reliable solutions to many different types of problems. Although a specific problem has been presented in this article, a coupled KBS architecture that employs a fuzzy optimization algorithm can be considered a general concept. This approach may be incorporated in applications to solve other, similar problems. Additionally, structuring the knowledge base within a Grafcet style network can help develop knowledge bases that are easily maintained and readily reviewed for explanations.

The application of a coupled KBS is demonstrated using the problem of hydrogen resource management. Specifically, heuristic methods and numerical methods are used to generate advice for refinery operators regarding the best hydrogen distribution. A portion of the KBS uses the current operating conditions to automatically specify a particular optimization problem. An optimization routine is then employed to solve for the optimal operating policy. Although this is a superior approach to merely mimicking operator procedures (Petti and Dhurjati, 1991), failure to consider uncertainty in the optimization model causes the approach to always choose a single "best" operating point. Advice may be generated recommending unnecessary changes when the current policy is close to the optimum. This unrealistic (and unintelligent) behavior results in diminished operator confidence in the system and wastes operator effort associated with insignificant changes.

It is therefore desirable to incorporate a method within the optimization to handle the uncertainty. This uncertainty is often associated with values that are merely opinions on preferred operation or are estimates of model parameters. The use of fuzzy optimization techniques is shown to be an effective method of dealing with such uncertainty. These techniques offer a means of interpreting the sensitivity information from the optimization. Furthermore, by taking a parametric approach to fuzzy optimization, the computational expense is relatively small while the representational power is greatly increased. This method uses standard programming techniques to solve the problem over the entire range of the uncertainty.

Using the results of the fuzzy optimization, the membership function describing the neighborhood of acceptable operating states is defined. In this manner, any operating point can be evaluated. By considering the current operating state, a decision can be made regarding the appropriateness of changing the current policy. Only, if significant improvement can be made will a recommendation to change the operating policy be advised. In this manner operator confidence can be maintained and wasted effort minimized.

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Notation

A = matrix ($m \times n$) of constraint coefficients, Eq. 1
a_j = jth row ($1 \times n$) of A, Eq. 13
b = vector ($m \times 1$) of constraint constants, Eq. 1
b_j = jth element of b, Eq. 13
C = fuzzy set defined by the constraints, Eq. 8
CCR = continuous catalytic reformer unit, Figure 1
CRU = catalytic reformer unit, Figure 1
c = vector ($1 \times n$) of objective function (z) coefficients, Eq. 1
D = fuzzy set defined by the decision region, Eq. 7
E_λ = λ-cut subset of a fuzzy set, Eq. 3
$f(\lambda)$ = optimal solution for the vector x as function of λ, Eq. 5
$g(\lambda)$ = optimal value of z as a function of λ, Eq. 6
H2U = hydrogen production unit, Figure 1
HCU = hydrocracker unit, Figure 1
HTU-i = ith hydrotreater unit, Figure 1
METH = methanol unit, Figure 1
m = number of constraints
NAPT = naphthalene unit, Figure 1
n = number of decision variables
PSA = pressure swing adsorption unit, Figure 1
SRU = sulfur recovery unit, Figure 1
x = vector ($n \times 1$) of decision variables, Eq. 1
x^* = optimal value of x, Eq. 5
x_j = jth element of x, Eq. 2
Z = fuzzy set defined by the objective function, Eq. 9
z = objective function (maximize), Eq. 1
z^* = optimal value of z, Eq. 6

Greek letters

λ = parameter characterizing the λ-cuts of a fuzzy set, Eq. 3
μ_A = membership function of fuzzy set A
ω = a subset of a fuzzy set, Eq. 3
τ_j = tolerance on the jth fuzzy constraint, Eq. 2

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